

A 3D finite element analysis (FEA) simulation of a mechanical part, likely a bracket or arm. The part is shown in a dark grey color with a green and yellow stress distribution on its internal surface. The background is a blue gradient with a wireframe mesh of a sphere and other mechanical components, suggesting a simulation environment.

# Modelovanje i simulacija procesa deformisanja

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## MODELIRANJE KONTAKTNOG TRENJA

**Kontaktno trenje** predstavlja otpor (abrazija i adhezija) relativnom kretanju dva tela u kontaktu između kojih deluje normalni napon. Otpor je proporcionalan čvrstoći slabijeg materijala i površini kontaktne površine!

Na veličinu kontaktnog trenja utiču:

**a) Vrsta procesa:** svaka vrsta obrade okarakterisana je određenim naponskim stanjem i veličinom kontaktne površine, a ti parametri utiču na veličinu trenja.

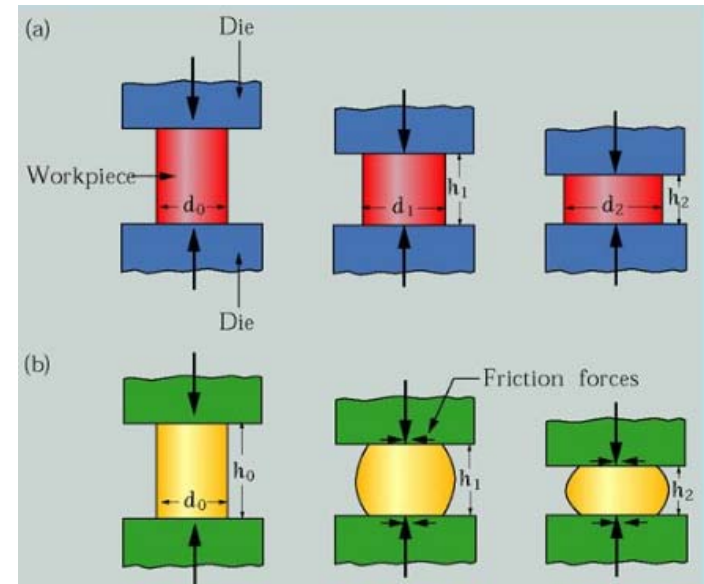
**b) Vrsta materijala alata i obratka:** uticaj vrste materijala je posebno izražen u slučaju deformisanja bez podmazivanja. Tada može doći do direktnog kontakta materijala alata i obratka i do međusobnog hladnog zavarivanja. Različiti materijali pokazuju različite tendencije ka hladnom zavarivanju.

**c) Temperatura obrade:** povećanjem temperature trenje raste do određenog trenutka da bi nakon toga opadalo. Kod čelika maksimalno trenje pojavljuje se na temperaturi između  $450^{\circ}$  i  $550^{\circ}\text{C}$ .

**d) Brzina deformacije:** povećanjem brzine deformacije smanjuje se kontaktno trenje.

**e) Podmazivanje:** podmazivanjem se veoma značajno utiče na smanjenje veličine trenja.

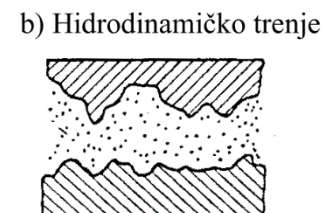
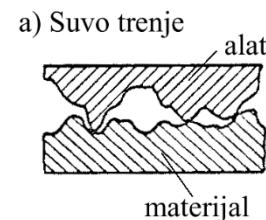
**f) Stanje kontaktnih površina alat–materijal:** što je površina alata kvalitetnija (manja hrapavost), trenje je manje. Zbog toga se alati za plastično deformisanje često poliraju.





## Vrste trenja

- **Suvo trenje** nastaje kada između dodirnih površina alata i materijala nema podmazivanja, tj. kada postoji direktan metalni kontakt.
- **Hidrodinamičko trenje** nastaje kada su dve kontaktne površine potpuno razdvojene mazivnim slojem. Ova vrsta trenja se ne javlja u obradama plastičnog deformisanja, sem u nekim specijalnim obradama.
- **Granično trenje** nastaje kada između alata i materijala postoji samo tanki sloj (film) maziva koji može – usled velikih lokalnih kontaktnih pritisaka – biti i prekinut, što dovodi do hladnog zavarivanja.
- **Mešovito trenje** je najčešći slučaj u praksi plastičnog deformisanja. To je trenje s elementima i graničnog i hidrodinamičkog trenja.





### **Uticaj trenja**

- mikrostrukturne promene deformisanog materijala
- habanje alata
- povećanje potrebne energije za deformisanje (50%)

### **Pravci triboloških istraživanja:**

1. definisanje mehanizma trenja preko zakona i teorija o trenju
2. razvoj metoda za ugradnju ovih definicija u modele deformacionih procesa i kvantitativnu procenu pokazatelja trenja.

- U opsegu sobnih temperatura, faktori koji imaju uticaja na kontaktno trenje su: mazivo, stanje površina, veličina deformacije i interakcija maziva i neravnina površine;
- Na povišenim temperaturama, značajni su faktori: veličina deformacije i stanje površina.



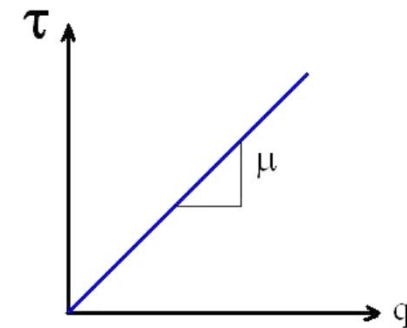
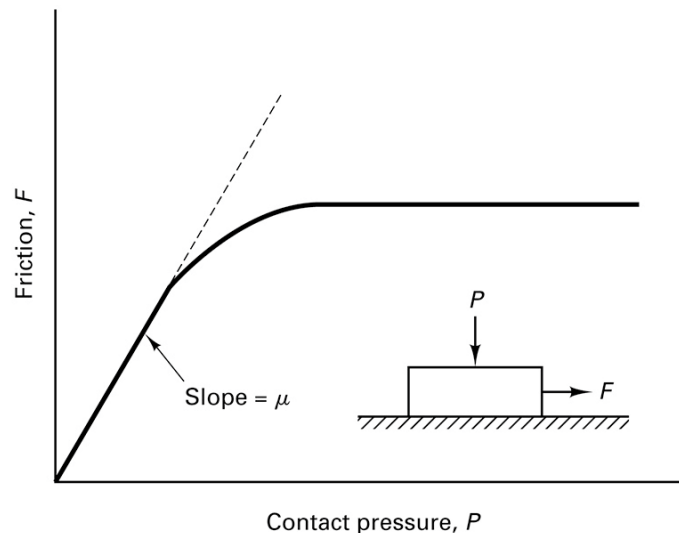
### 1. Amonton-ov (1699) i Coulomb-ov (1785) zakon

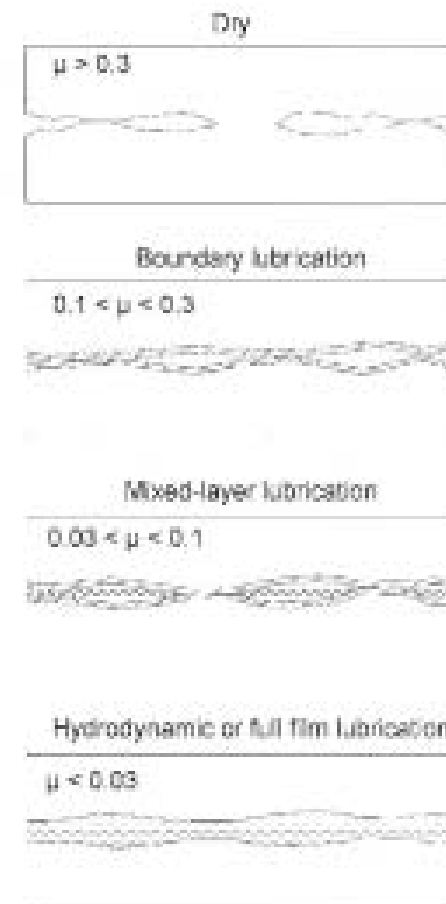
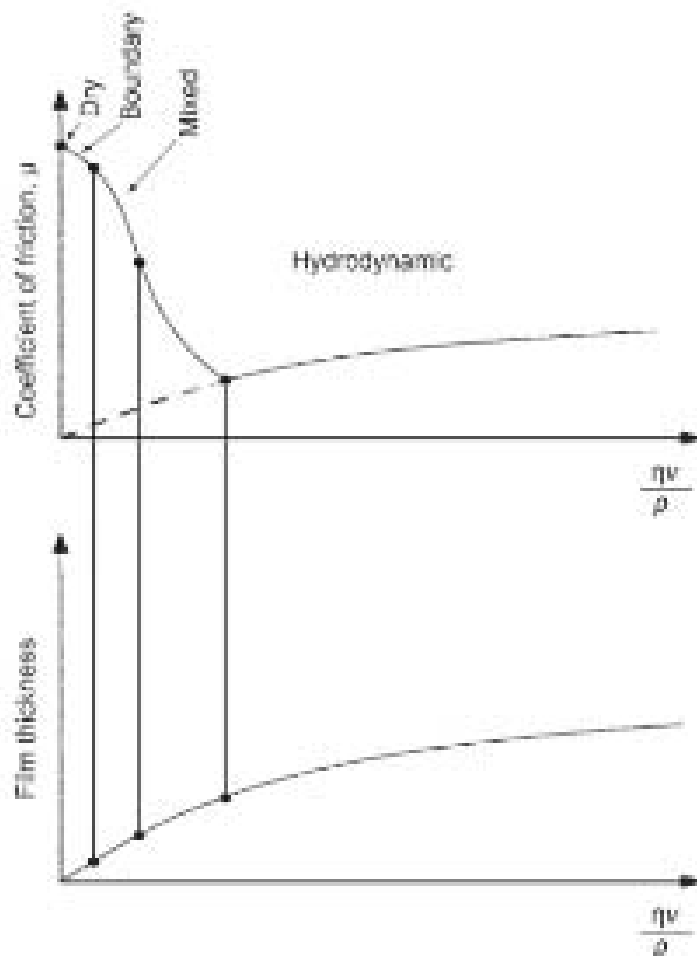
$$\tau_k = \mu \cdot \sigma_n = \mu \cdot p \quad 0 \leq \mu \leq 0.577$$

$$\tau_k = \mu \cdot K$$

- topla obrada  $\mu = 0,4-0,5$
- hladna obrada bez podmazivanja  $\mu = 0,2-0,3$
- hladna obrada sa podmazivanjem  $\mu = 0,08-0,15$

$$\mu = \frac{F}{P} = \frac{F/A}{P/A} = \frac{\tau_{friction}}{\sigma_{normal}}$$







## 2. Zakon konstantnog trenja (Orowan 1946)

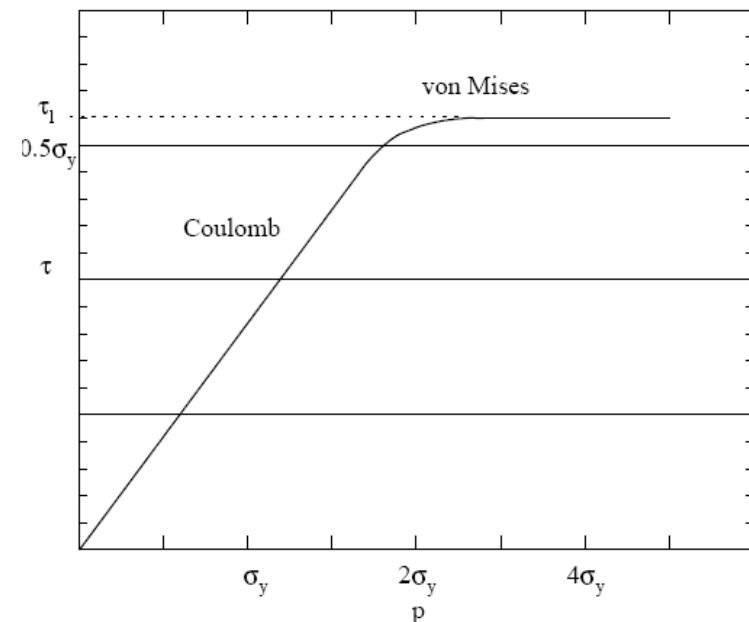
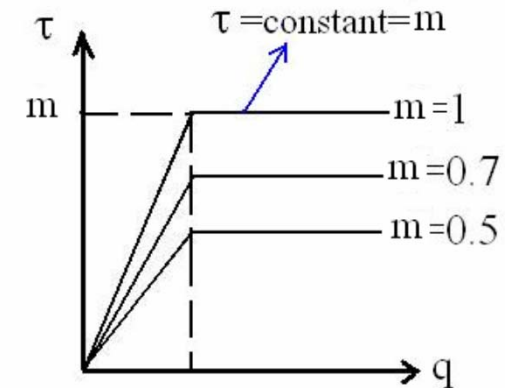
$$0 < \tau_k < \tau_{\max} \Rightarrow \tau_k = m \cdot \tau_{\max} \quad 0 \leq m \leq 1$$

$m$  - shear friction factor (faktor smicanja-trenja)

$$\tau_k = m \tau_{\max} = m \frac{\sigma_1 - \sigma_2}{2} \Leftrightarrow m \frac{K}{2} \beta = \mu \cdot K$$

$$\mu = m \frac{2}{\sqrt{3}} \frac{1}{2} = \frac{m}{\sqrt{3}}$$

$$\mu_{\max} = \frac{m_{\max}}{\sqrt{3}} = \frac{1}{\sqrt{3}} = 0,577$$







### 3. Opšta (adheziona – Bodwen i Tabor 1942) teorija trenja – Opšti model (Wanheim - 1974, Bay and Petersen - 1976)

Napon trenja ( $\tau_k$ ) :

- ☐ normalni pritisak
- ☐ topografija površine
- ☐ dužina klizanja - kontakta
- ☐ viskoznost i kompresibilnost lubrikanta

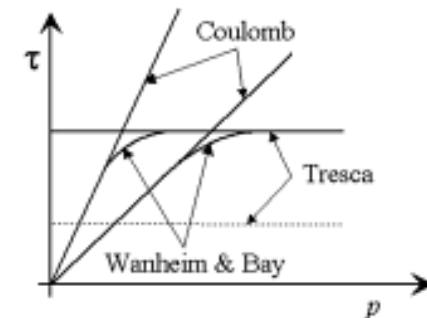
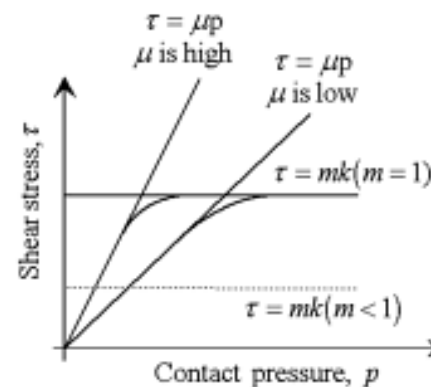
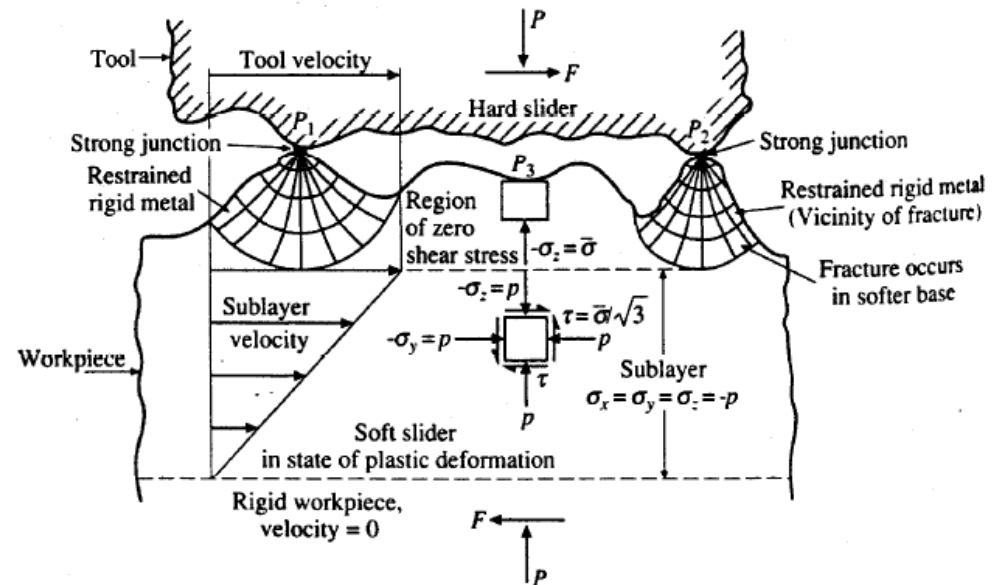
#### Teorija linija klizanja

$$\tau_k = f \alpha K$$

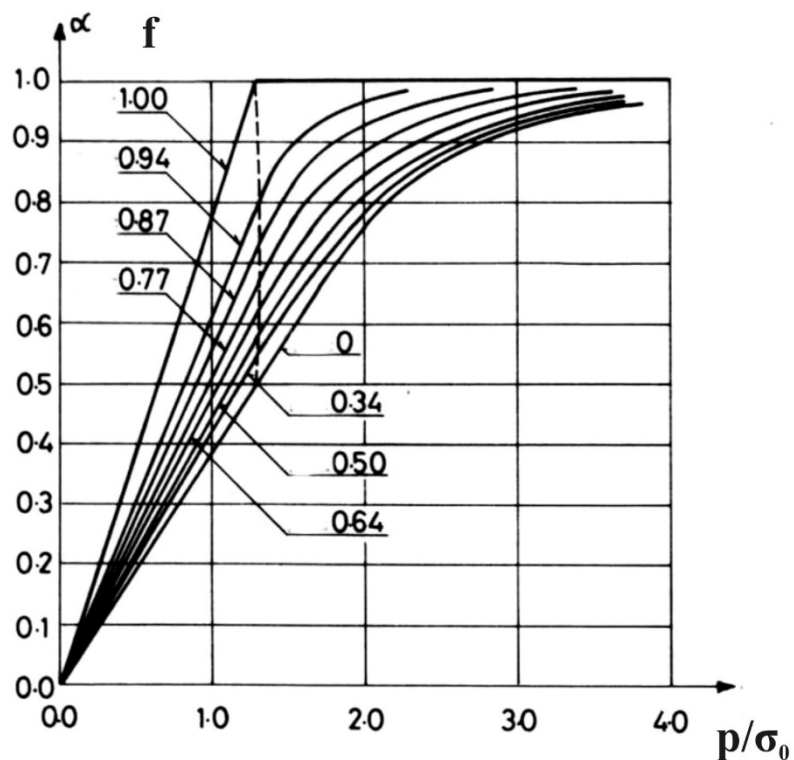
$f$  – faktor trenja ( $0 \leq f \leq 1$ )

$\alpha$  - odnos realne zone kontakta  
i računske površine kontakta

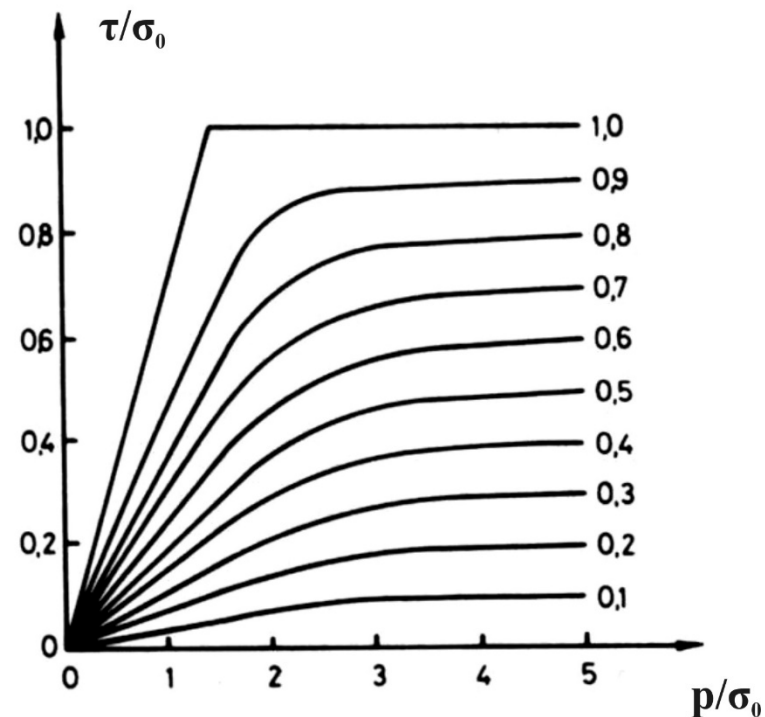
$K$  – napon tečenja



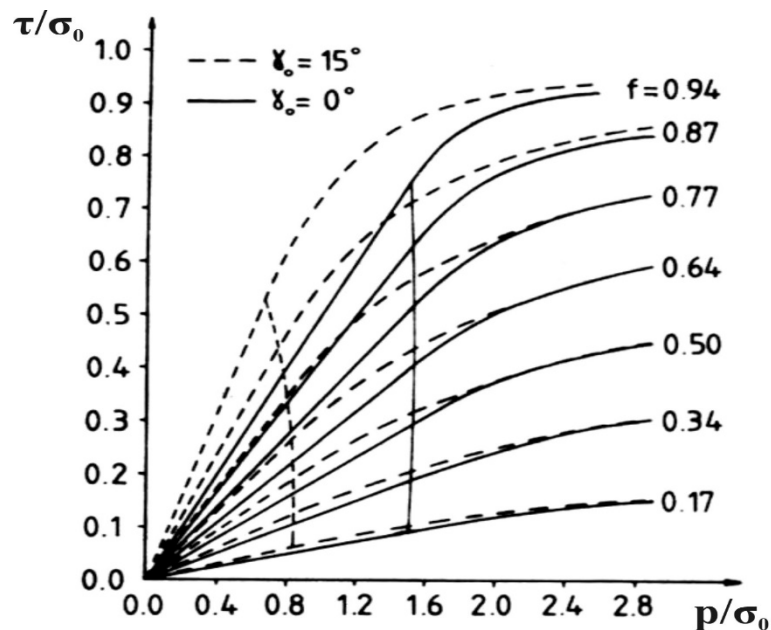




*Realna zona kontakta kao funkcija pritiska i faktora trenja (Bay 1976)*



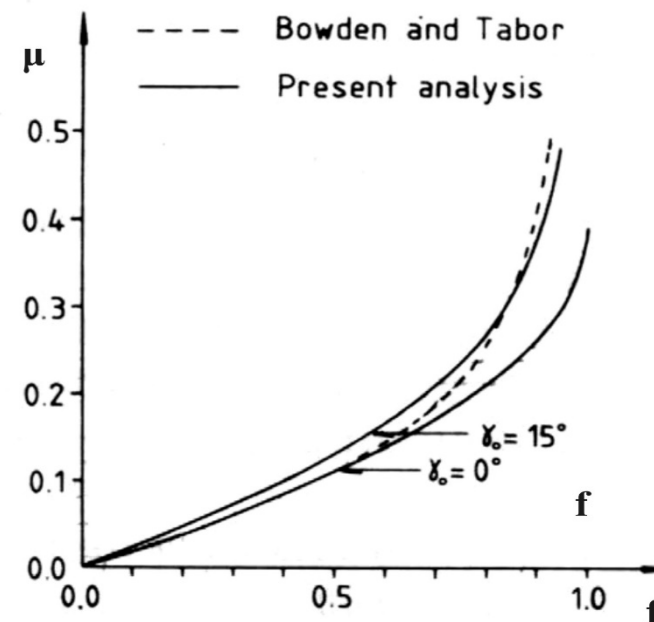
*Tangencijalni napon kao funkcija pritiska i faktora trenja (Wanheim 1974)*



*Granica proporcionalnosti tangencijalnog napona i normalnog pritiska (Bay 1987)*

$$\frac{p'}{\sigma_0} \approx 1.5, \quad \gamma_0 = 0^\circ, \quad f < 0.9$$

$$\frac{p'}{\sigma_0} \approx 0.8, \quad \gamma_0 = 15^\circ, \quad f < 0.9$$



*Koeficijent trenja kao funkcija faktora trenja i početnog ugla neravnine (Bay 1987)*

$$\mu = \frac{f}{\left(1 + \pi/2 + \arccos - 2\gamma_R \sqrt{1-f^2}\right)} - \text{Bay}$$

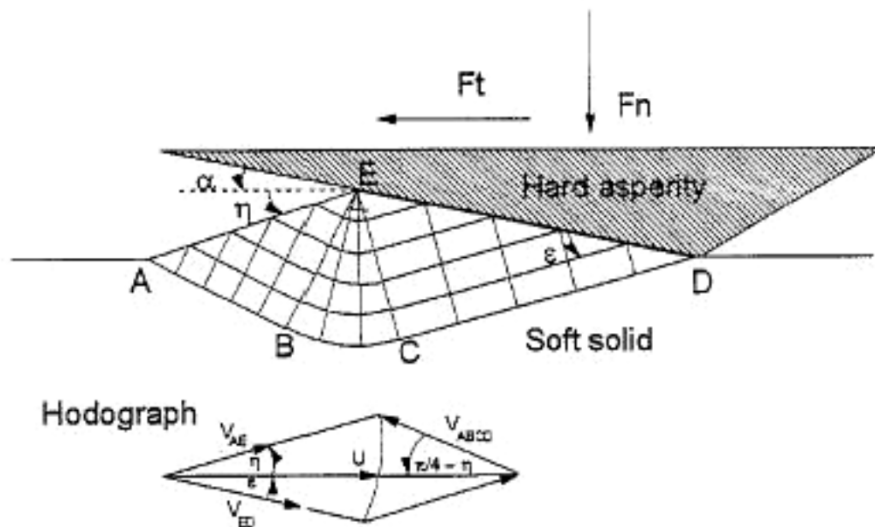
$$\mu = \frac{f}{\sqrt{27(1-f^2)}} - \text{Bowdwen i Tabor}$$



Friction model	Friction stress distributions	Main assumptions and applications	Authors, (year)
$\tau = \mu \cdot p$		Friction model Friction stress $\tau$ is directly proportional to local normal pressure $p$ . It is mainly used for cold metal forming due to its simplicity.	Von Karman, 1925 Kunogi, 1924 Kudo, 1960
$\tau = m \cdot k$		Dry slipping occurs over the whole tool/workpiece interface. $k = \sigma_0 / \sqrt{3}$ is the shear flow stress, and $\sigma_0$ is the yield stress.	Orowan, 1946
$\tau = \frac{K}{2}$		Sticking occurs over the whole interface between tools and workpiece $K = 1.15 \sigma_0$ .	Sims, 1954 Nadai, 1939
$\tau = \eta \frac{dv_x}{dy}$		Viscous slipping friction proportional to relative velocity of slip, occurs over the whole interface between tools and workpiece.	Nadai, 1939
Area I: $\tau = \frac{K}{2}$ Area II: $\tau = \mu \cdot p$		The interface is divided into two zones: (I) sticking occurs at the central zone whose centre is the neutral point; (II) dry slipping occurs at the edge zone when frictional stress is less than yield stress in shear.	Orowan, 1943
Area I: $\tau = \tau_{II} \frac{r}{OA}$ Area II: $\tau_{II} = \frac{K}{2}$ Area III: $\tau = \mu \cdot p$		A zone of restricted plastic deformation exists in the middle of the sticking zone. The tool/workpiece interface is divided into three zones: (I) the stick zone, (II) the drag zone, and (III) the slip zone.	Tselikov, 1958 Unkssov, 1961
Area I: $\tau = \tau_A \frac{r}{OA}$ Area II: $\tau = f \alpha k$ Area III: $\tau = \mu \cdot p$		Three zones are similar to the model of Tselikov and Unkssov: (I) the central sticking zone; (II) the sliding zone and (III) the homogeneous deformation zone.	Bay and Gerved, 1984



## Torrance's Friction Model (Slip line theory)



$$F_T = [A \sin \alpha + \cos(2\varepsilon - \alpha)] \cdot \overline{ED}k$$

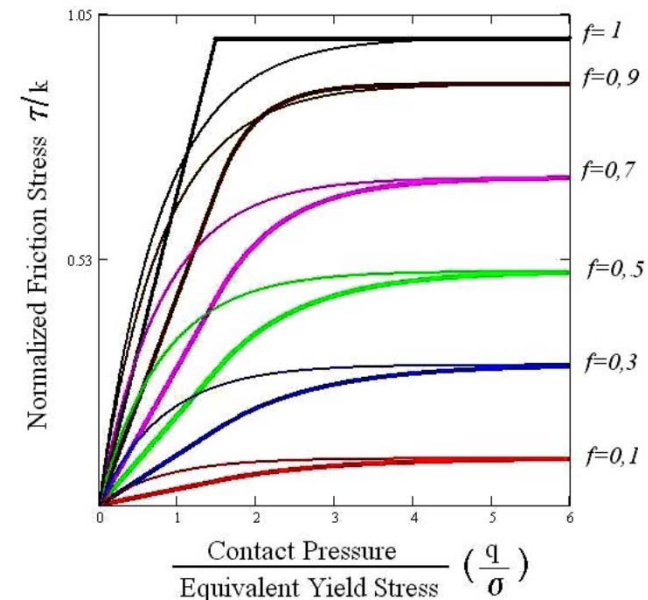
$$F_n = [A \cos \alpha + \sin(2\varepsilon - \alpha)] \cdot \overline{ED}k$$

$$\mu = \frac{F_T}{F_n}$$

$$A = 1 + \frac{\pi}{2} + 2\varepsilon - 2\eta - 2\alpha \quad , \quad 2\varepsilon = \arccos(i) \quad \text{and} \quad i = \frac{\tau}{k}$$

## Levanov's Friction Model (FORM2D )

$$\frac{\tau}{k} = f \left[ 1 - \exp \left( -1.25 \left( \frac{q}{\sigma_0} \right) \right) \right]$$





## TRENJE I MKE

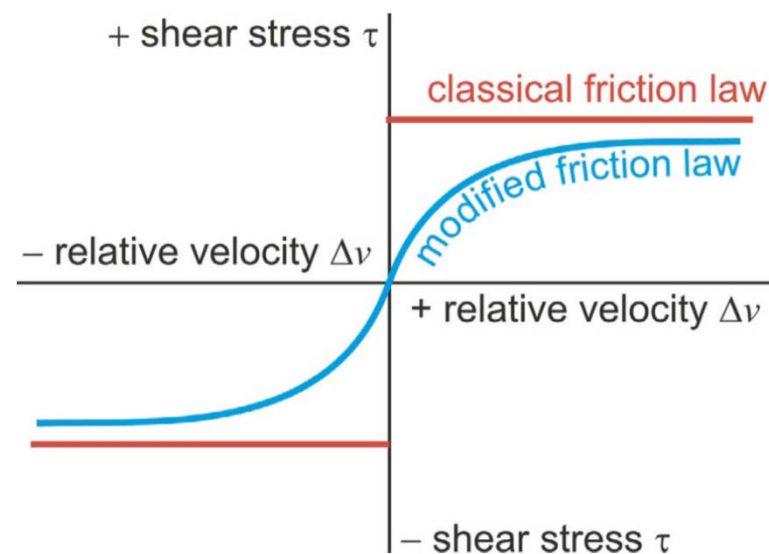
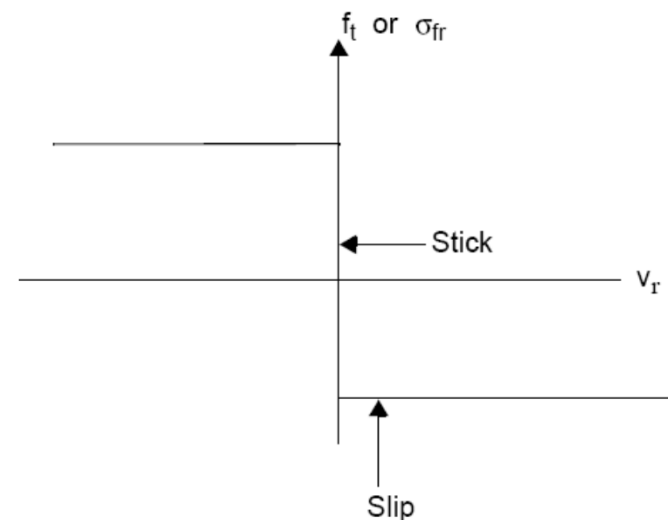
$$\Pi = \int_V \bar{\sigma} \dot{\epsilon} dV + \int_V \frac{1}{2} K \epsilon_v^2 dV + \int_{S_c} \left( \int_0^{|u_r|} \tau_n du_r \right) dS$$

$$\pi_c = \int_{S_c} \left( \int_0^{|v_r|} \tau_f dv_r \right) dS \quad \tau_f = -\mu \sigma_n \frac{v_r}{|v_r|}$$

$$|\tau_{\text{Friction}}| = \mu \cdot \sigma_{\text{normal}} \cdot \left( \frac{\Delta v}{\omega + |\Delta v|} \right)$$

$$|\tau_{\text{Friction}}| = \mu \cdot \sigma_{\text{normal}} \cdot \frac{2}{\pi} \tan^{-1} \left( \frac{\Delta v}{\omega} \right)$$

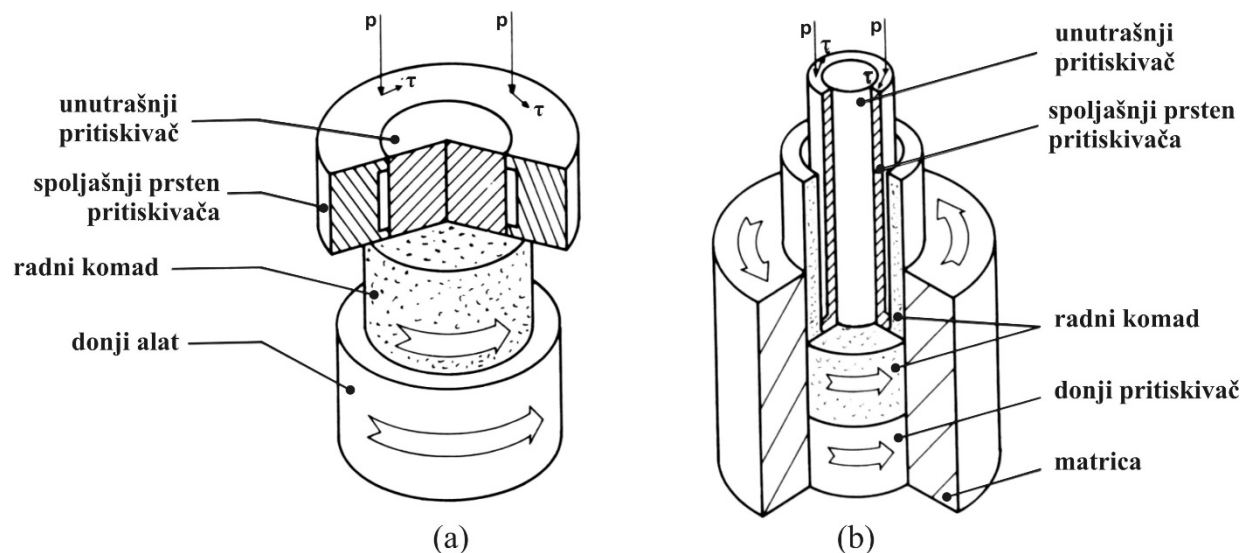
$\omega$  - proizvoljna konstanta  
(vrednost relativne brzine, kada dođe do klizanja)





## METODE ODREĐIVANJA POKAZATELJA KONTAKTNOG TRENJA

**Direktne metode** određivanja kontaktnog trenja merenjem (eksperimentalno) lokalne sile trenja, ili nekih drugih pokazatelja, u zavisnosti od primenjene metode i vrste obrade



*Test pritiskivanja-uvijanja (compression-twist tests)*

Koeficijent trenja pri valjanju

$$\mu = \frac{T}{2P \cos(\alpha/2)} + \operatorname{tg}(\alpha/2), \quad \alpha = \sqrt{\Delta h/R}, \quad \Delta h = h_0 - h_1$$

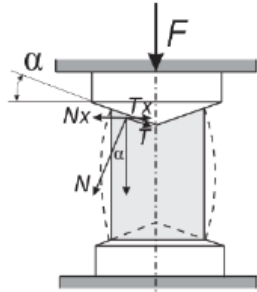
Faktor trenja pri vučenju žice

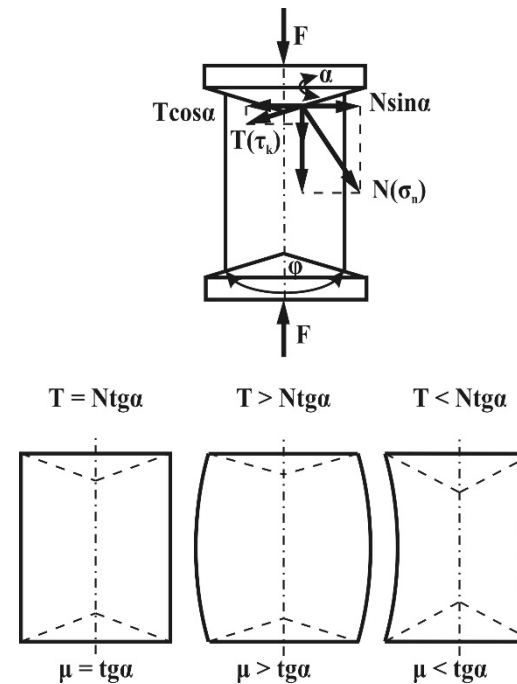
$$m = \left( \frac{2}{3} \right) \frac{\alpha_{opt}^2}{\ln(R_0/R_f)}$$





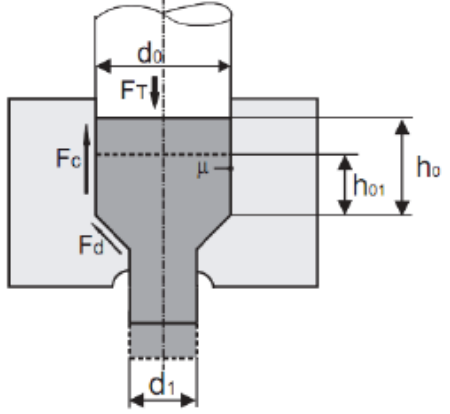
**Indirektne metode** određivanja kontaktnog trenja praćenjem određenih geometrijskih veličina uzoraka u uslovima podeljenog tečenja materijala (*divided flow tests*).

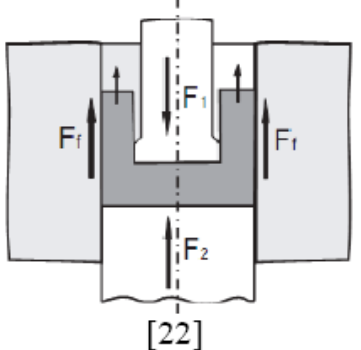
Upsetting of cylinder by using conical compression platens	
Schema	 <p>[29]</p>
Principle	<p>When horizontal components of normal and friction force are in balance (<math>N_x = T_x</math>) cylinder deforms uniformly, no barrelling takes place. In this case <math>\mu = \tan \alpha</math>. When the friction component is higher (<math>T_x &gt; N_x</math>), cylinder barrelling occurs and in case that horizontal component of normal force prevails (<math>N_x &gt; T_x</math>) the end faces spread.</p>
Application field	<p>This test is convenient for frictional study and lubricant evaluation in bulk metal forming. Drawback of this method is requirement for a large number of conical dies with different angles and corresponding specimens.</p>



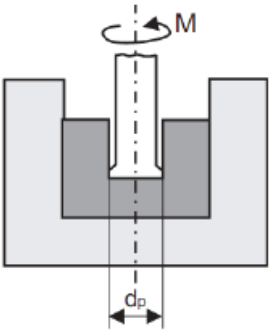


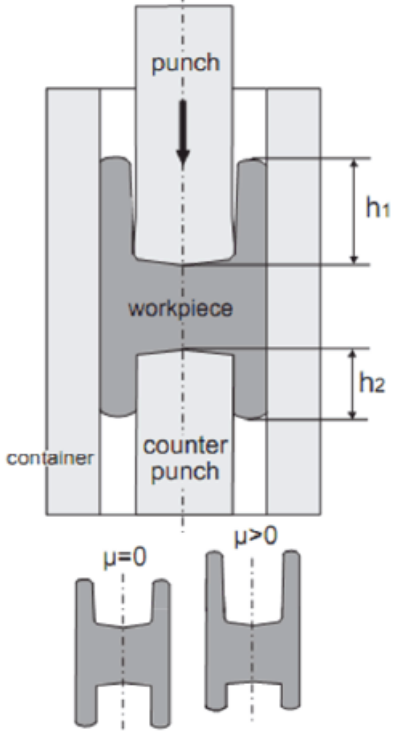


Forward bar extrusion	
Schema	 <p>[18]</p>
Principle	<p>In this test cylindrical billet is extruded through conventional forward extrusion die. During observed interval, height <math>h_0</math> decreases to <math>h_{01}</math> and load <math>F_T</math> decreases for <math>\Delta F_T</math>. Load reduction <math>\Delta F_T</math> is a function of only <math>F_C</math> (other force components during this interval remain constant). Friction coefficient is a function of <math>\Delta F_T</math>, die geometry and material yield stress.</p>
Application field	<p>This method can be used where more severe strains prevail. Forward bar extrusion test requires load measurement and knowledge of material yield stress.</p>

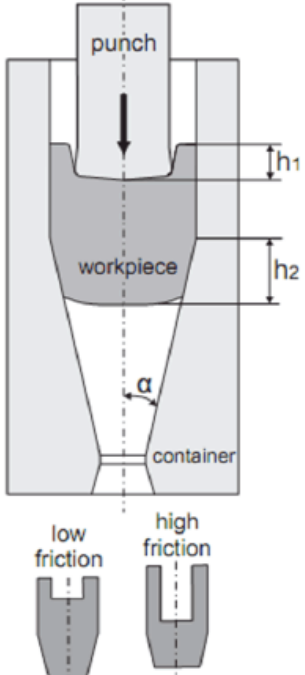
Backward cup extrusion	
Schema	 <p>[22]</p>
Principle	<p>In backward cup extrusion punch forces the billet's material to flow sideways through the gap between the punch head and container. By measuring forces <math>F_1</math>, <math>F_2</math> and <math>F_f</math> and by knowing tool/die geometry and material yield stress, friction can be calculated.</p>
Application field	<p>This test is most suitable for processes with high effective strains. Loads measurements, as well as material yield stress are needed.</p>

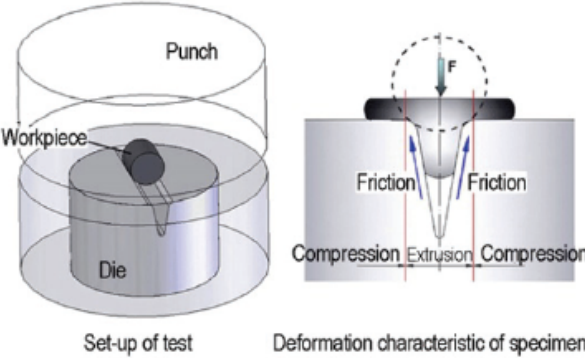


Backward extrusion with a twist	
Schema	 <p>[23]</p>
Principle	Billet is first backward extruded and then the punch is rotated while the die is kept stationary. By introducing another punch (with different punch land), both momentums are measured and friction is calculated.
Application field	This test is friction evaluation in processes with low strains. Two different punches and mechanism for punch rotation are needed

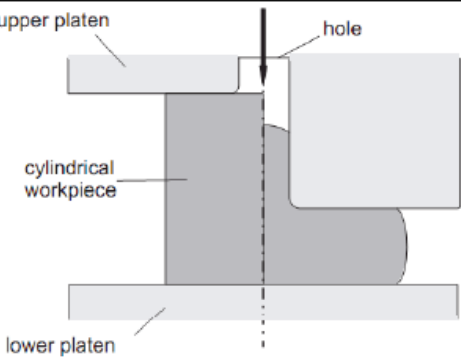
Backward – forward hollow extrusion	
Schema	 <p>[18]</p>
Principle	In this method a cylindrical billet is extruded in both upward and downward direction. Ratio $h_1/h_2$ is friction sensitive, i.e. the higher the friction, the more material flows upwards.
Application field	This test is for friction assessment in processes with high pressures and deformations.

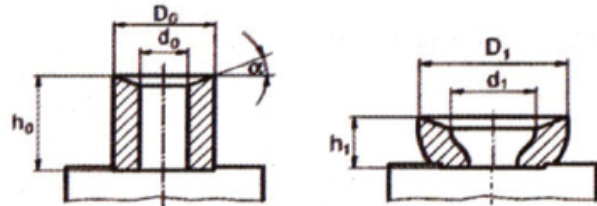


Combined forward – backward extrusion	
Schema	 <p>[18]</p>
Principle	<p>Cylindrical billet is extruded in both forward and backward direction. Therefore material flows through the opening at the bottom of the die and through the gap between the punch and die. The higher the friction, the more material flows backward. Ratio <math>h_1/h_2</math> is the indicator of friction magnitude.</p>
Application field	<p>Both backward and forward deformation occurs in this test. In this test large strains and pressures occur.</p>

T – Shape compression	
Schema	 <p>[25]</p>
Principle	<p>In this test cylindrical specimen is placed at the die with a V – groove. During compression by flat punch, material flows in two directions: downwards in the groove and sideways between the tools. The amount of material that flows in the groove is friction sensitive. By FE simulation friction calibration curves are generated.</p>
Application field	<p>Oil is easily applied in the test by filling the groove. This test is for friction assessment in processes with severe deformations (both extrusion and compression). Specimens with different diameters can be used with the same tools.</p>

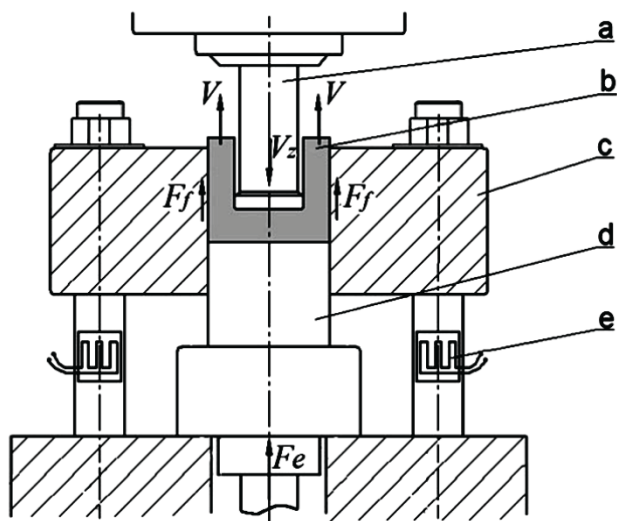
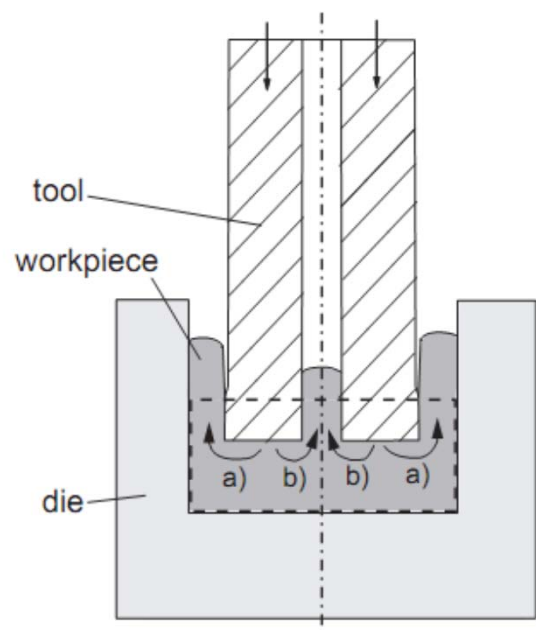


Open die backward extrusion test	
Schema	 <p>[26]</p>
Principle	<p>Cylindrical billet is compressed by a punch with a hole and flat die. Material flows both upwards, through the punch hole and horizontally, between the punch and die. Height of the billet at the end of the process is the indicator of friction magnitude. Friction calibration curves are obtained by FE analysis.</p>
Application field	<p>In this test it is possible to vary billets' initial geometry without changing tools. This test can be used to obtain friction magnitude in processes where large deformations prevail.</p>

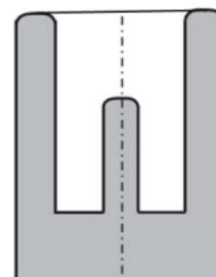
Conical Tube – upsetting test	
Schema	 <p>[28]</p>
Principle	<p>In this test a cylinder with a drilled hole is upset by plane die (at the bottom) and conical die (at the top). Friction is evaluated by measuring final height and outside diameter of the billet.</p>
Application field	<p>There is no sticking zone (unlike in ring compression test). However, geometries of the tools are more complex. This method is for friction evaluation in processes where low strains occur.</p>



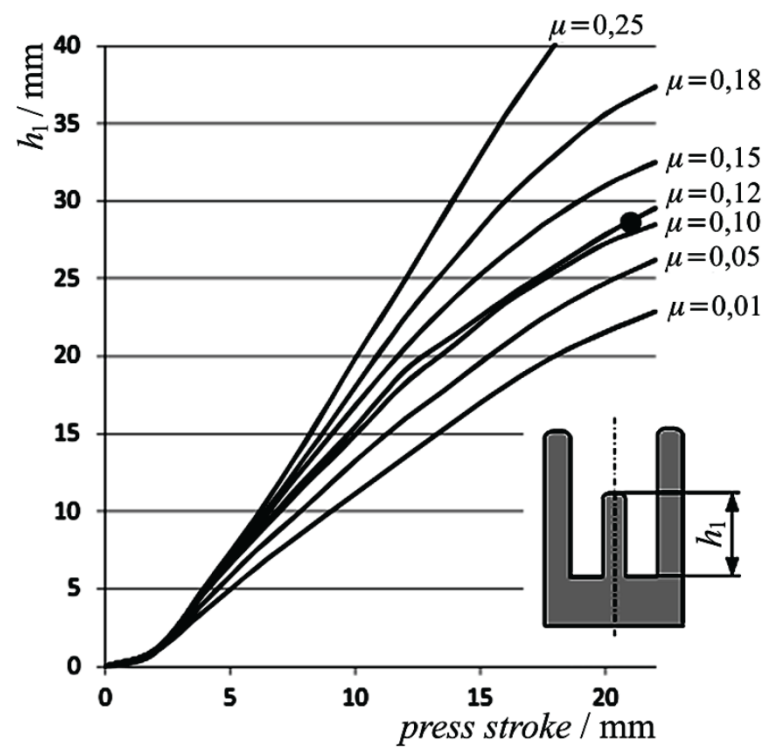
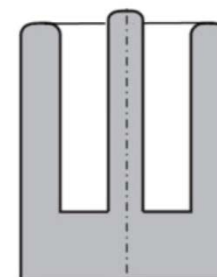
Tapered – plug penetration test	
Schema	<p>[27]</p>
Principle	Tapered – plug is penetrated into hollow billet made from titanium alloy. Tapered – plug penetration load is measured and used as indicator of lubricant performance.
Application field	This test is used for comparison of different lubricants.



low friction



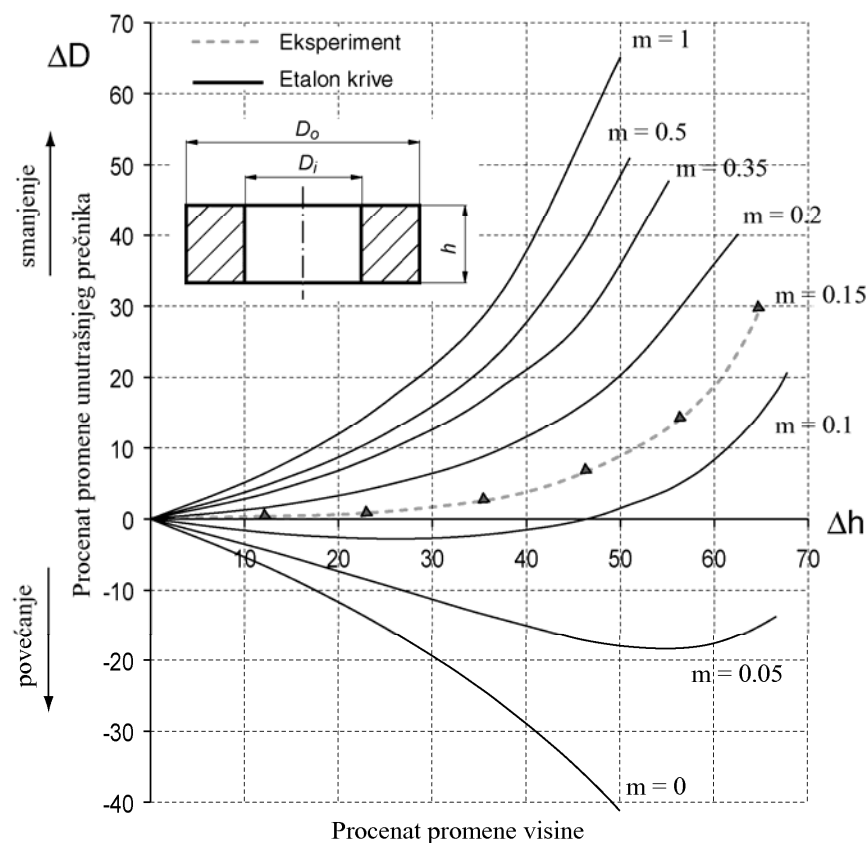
high friction



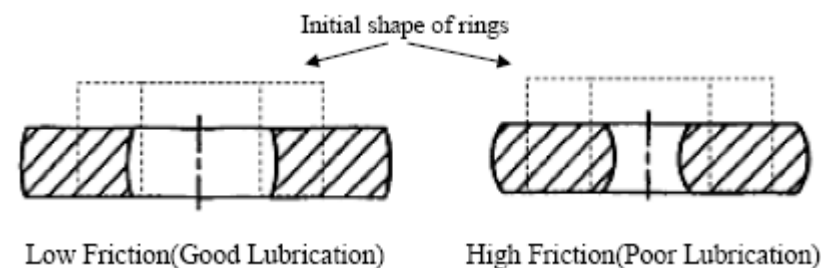




## Sabijanje prestena - Ring test



$$D_o : D_i : h = 6 : 3 : 2$$



Reduction  
in height [%]

7.2%

22.2%

40.0%

Decrease  
in I.D. [%]

-1.48%

-2.96%

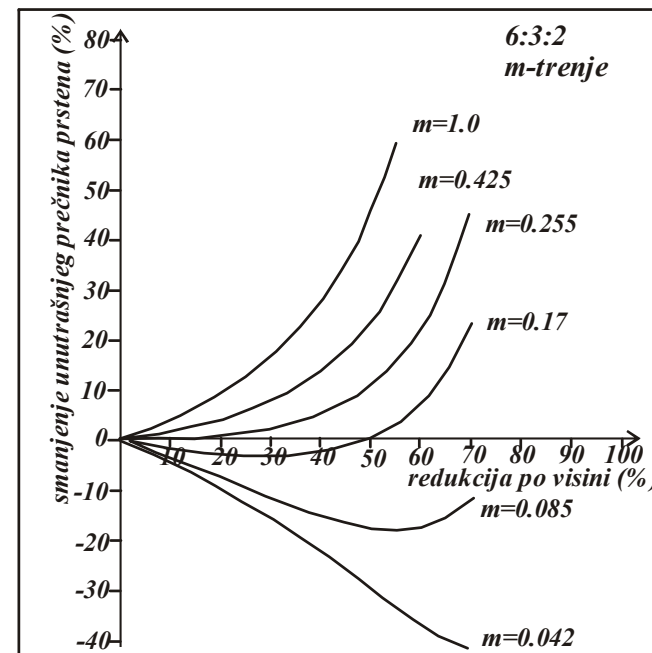
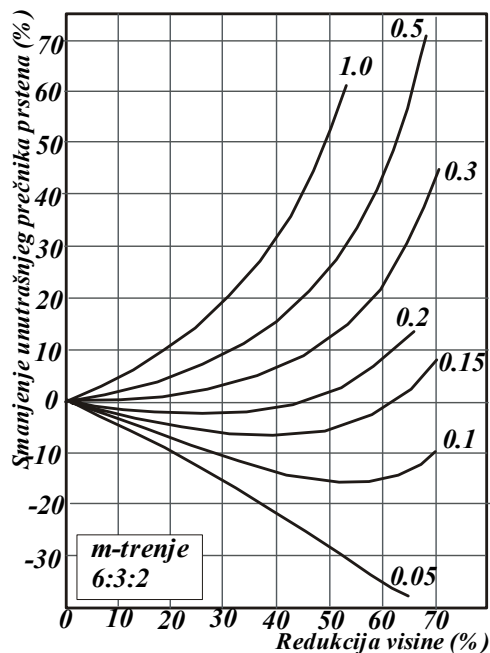
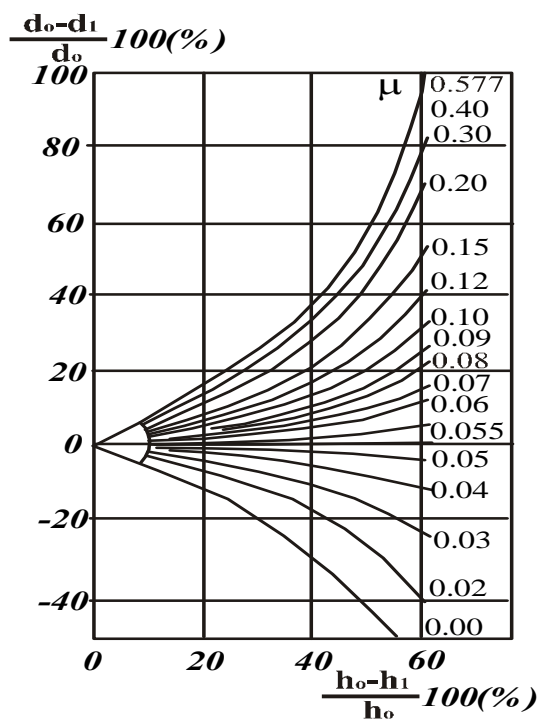
-1.85%



$$\varepsilon_h = \frac{h_0 - h_{tr}}{h_0}$$

$$\varepsilon_d = \frac{D_0 - d}{D_0}$$



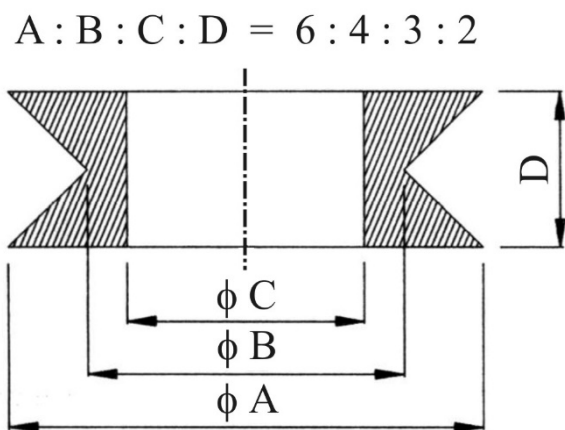


Male-Cocroft kalibracione krive 6:3:2, za  $\mu$ -trenje i  $m$ -trenje

Kalibracione krive 6:3:2, za  $m$ -trenje  
(Danckert 1988)

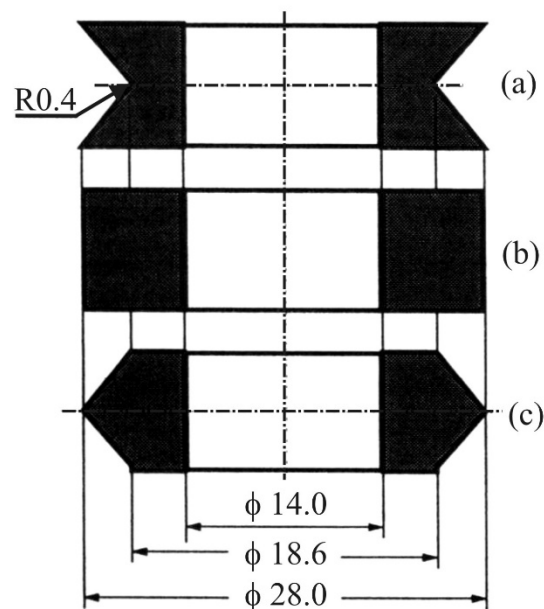


## KOMPLEMENTARNA *RING TEST* METODA



*Geometrija prstena za komplementarnu ring test metodu (Petersen 1998)*

$$\mu = \frac{A}{p} \rightarrow \tau = \mu p = A, \quad (0 \leq A \leq k_{\max})$$



*Geometrije prstena 6:4:3:2 (a) mali pritisci, (b) normalni pritisci, (c) veliki pritisci*